MA 3232 - Numerical Analysis
Sample Final Exam

Instructions: Work all problems. Read the problems carefully. Show appropriate work, as partial credit will be given. Three pages ($8\frac{1}{2}$ by 11) of notes (both sides) and Mathematics Department "Blue Books" of tables permitted.

---

1. (30 points) Consider the table of data:

x	$f(x)$	$\Delta f(x)$
0.00	0.0000	0.1511
0.15	0.1511	0.1582
0.30	0.3093	0.1738
0.45	0.4831	0.2010
0.60	0.6841	0.2475
0.75	0.9316	0.3286
0.90	1.2602	

a. Estimate $f(0.33)$ using the *most appropriate* second-degree Newton-Gregory forward polynomial.

b. Estimate the error in your answer to part.a.

c. How would your answers to parts a. and b. above have changed had you used a second-degree Lagrange polynomial on the same data points?

Exam Continues on Next Page

2. (40 points) Consider the table of data:

x	$f(x)$
0.00	0.0000
0.15	0.1511
0.30	0.3093
0.45	0.4831
0.60	0.6841
0.75	0.9316
0.90	1.2602
1.05	1.7433
1.20	2.5722

Estimate $\int_0^{1.2} f(x)dx$ using *Simpson's rule* and estimate the error in your answer.

---

3. (30 points) a. Using second order, centered differences, replace the differential equation

$$4u'' - u' + u = x$$

by an approximate expression involving terms such as $u_i,\ u_{i-1}$, etc., where $u_i = u(x_i)$.

b. Why is extrapolation (e.g. Romberg) normally *not* used to estimate the error in single-step ordinary differential equation solvers?

c. A finite difference approximation, when applied to a known function, produces errors that vary according to the step size as follows:

h	*error*
0.800	0.1396
0.400	0.0081
0.200	0.0005

What is the apparent order of the error for this method?

Exam Continues on Next Page

4. (50 points) Consider the initial value problem

$$y' = \frac{x}{y^2+1}$$

$$y(1) = 1$$

with the following tables of values:

x	y	y'
1.00	1.000	0.5000
1.25	1.132	0.5479
1.50	1.272	0.5730

a. Estimate y(2.0) using *two* steps ($h = 0.25$) of the third-order Adams-Moulton-Bashforth algorithm given by:

$$y^p_{n+1} = y_n + \frac{h}{12}\left[23f_n - 16f_{n-1} + 5f_{n-2}\right]$$
$$y^c_{n+1} = y_n + \frac{h}{12}\left[5f^p_{n+1} + 8f_n - f_{n-1}\right]$$

where

$$f^p_{n+1} = f(x_{n+1}, y^p_{n+1})$$

b. Would a higher-order Adams-Bashforth-Moulton method have been more appropriate *for this problem*? (Briefly *explain* your answer!)

---

5. (30 points) a. Use two iterations of Newton's method to estimate the root of:

$$f(x) = 4xe^{-x} - 1$$

which lies between $x = 0$ and $x = 0.5$.

b. Estimate the error in your answer for part.a.

c. Would the Secant method have been preferable to Newton's method for this problem? (*Briefly* explain your answer.)

---

6. (20 points) a. Consider the difference approximation:

$$f'(x_n) \doteq \frac{f_{n+1} - f_{n-1}}{2h}$$

Using Taylor Series, show that the error of this approximation is $\mathbf{O}(h^2)$.

b. Is this a forward, backward or central approximation?
